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LETTER TO THE EDITOR

Classification of the dynamical symmetries in the extended interacting boson model

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Abstract. All the possible dynamical symmetries associated with the interacting boson model with s, d and g bosons (gIBM) are classified.

One of the ways of extending the original interacting boson model (Arima and Iachello 1976, 1978, 1979) consists of the incorporation of the g boson into the model. Thereby the usual U(6) symmetry group is replaced by U(15). Several investigations in this domain have been carried out, some using a perturbative approach (Sage and Barret 1980, Van Isacker *et al* 1982) and others by considering the SU(3) dynamical symmetry present in the gIBM (Ratna Raju 1981, Goldfarb 1981, Wu 1982, Wu and Zhou 1984, Yoshima 1985). Although the IBM itself already allows for three kinds of dynamical symmetries, i.e. SU(3), SU(5) and O(6), nearly all considerations and efforts in support of a g boson incorporation have been concentrated upon the limit of rotational nuclei for which the pertinent reduction chain is $U(15) \supset SU(3) \supset O(3)$. The reduction $U(15) \supset SU(3)$ can be accomplished by rules set out by Elliott (1958). A further and more thorough discussion of the mathematical properties of the SU(3) dynamical symmetry has been given by Sun *et al* (1983). These authors also consider a second dynamical symmetry associated with the chain $U(15) \supset SU(5) \supset O(5) \supset O(3)$, which in a certain sense opens up the possibility for the gIBM to also describe vibrational nuclei.

It is natural to ask whether other dynamical symmetries exist in the U(15) model. This question has recently been answered affirmatively by Kota (1984) using physical arguments. It is the aim of this letter to further investigate the completeness of Kota's results by means of elementary representation theory. Details concerning the labelling of states, explicit reduction formulae, Casimir operators and boson operator realisations of the occurring subalgebras shall be treated elsewhere.

The problem of finding all dynamical symmetries of the U(15) model is equivalent to the construction of all physical group-subgroup inclusion chains starting from U(15) and ending at the physical subgroup O(3). By physical it is understood that the fifteen-dimensional irreducible representation (irrep) of U(15) decomposes through the chain $U(15) \supset G_1 \supset G_2 \supset \ldots \supset G_k \supset O(3)$ into the O(3) irreps (0), (2) and (4) with respective dimensions one, five and nine. Hence the generators of U(15) and of all

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subgroups in the chain may be realised as O(3) tensor operators acting in a fifteen-dimensional space specified by $l = 0$, 2 and 4. In boson terminology this means that the U(15) Lie algebra consists of all the possible couplings of an s, d or g boson creation operator with an s, d or g boson annihilation operator.

For our purpose the most important observation is that each of the subgroups G_i in the chain must contain irreps with dimensions adding up to fifteen and hence being fifteen, fourteen and one, ten and five, nine and six or nine and five and one, respectively. As a first example, we study the simple Lie algebras having a fifteen-dimensional irrep and with a rank inferior to that of SU(15). Using the standard tables of McKay and Patera (1981) and Wybourne (1970), it is readily verified that they are the Lie algebras of the respective groups O(3), SU(3), SU(4) ≈ O(6), SU(5), SU(6) and O(15). As a next step we verify for each of these algebras whether their fifteen-dimensional irrep is decomposable into the O(3) irreps (0), (2) and (4). With that aim we again use the tables established by McKay and Patera (1981).

To quote an example, we notice that in the decomposition chain O(15) ⊃ O(14) ⊃ O(5) ⊃ O(3), the fifteen-dimensional O(15) irrep (1000000) reduces first into the O(14) irreps (1000000) and (0000000) of dimension fourteen and one respectively. The former further reduces into the O(5) irrep (2, 0) which in turn reduces into the O(3) irreps (2) and (4). The latter trivially reduces into the O(3) irrep (0). It follows, moreover, that the O(14) and O(5) generators can be realised as O(3) tensor operators acting in a space specified by $l = 2$ and 4, or equivalently by means of d and g boson creation and annihilation operators only. By inspection of the tables, it turns out that all other chains starting at O(15) are non-physical in the present meaning.

Starting at SU(6), SU(5) or SU(3) we also find in each case one physical chain, whereas for SU(4) no such chain can be established. The explicit structure of the physical chains obtained so far has been graphically represented in figure 1.

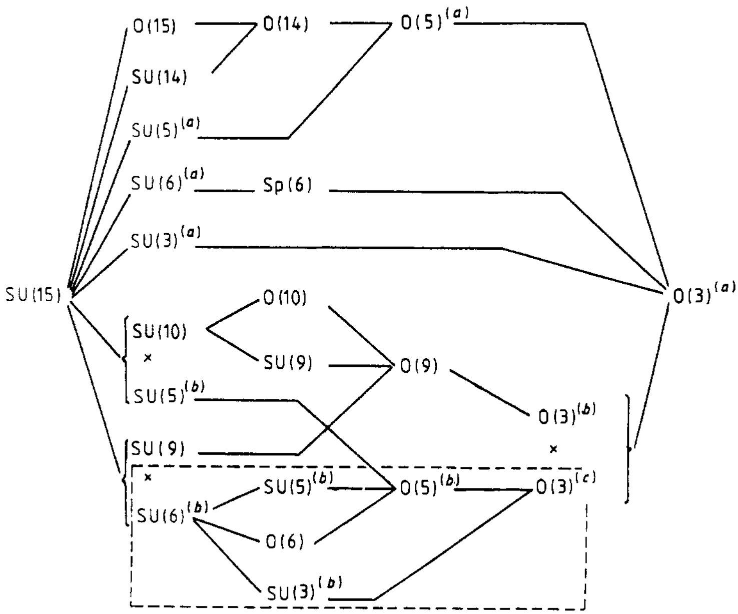

Figure 1. Classification of the dynamical symmetry groups in the U(15) extended IBM and their consecutive decomposition into the physical O(3) group.

The second case, which we discuss in more detail, is that of the Lie algebras possessing both a ten-dimensional and a five-dimensional irrep. The simple Lie algebras satisfying this condition are SU(5) and O(5) but none of these leads to a physical chain. However, at this point we may no longer disregard semi simple Lie algebras. Indeed, $SU(15) \supset SU(10) \times SU(5)$, whereby the fifteen-dimensional irrep reduces to a ten-dimensional irrep $(100000000) \times (0000)$ and a five-dimensional irrep $(000000000) \times (1000)$ of $SU(10) \times SU(5)$. Consequently, the $SU(10) \times SU(5)$ generators are the sum of the SU(10) generators realised by means of s and g boson operators and of the SU(5) generators realised by means of only d boson operators. In order to study further the chains starting at $SU(10) \times SU(5)$, it suffices to analyse separately the SU(10) and SU(5) algebras, whereby it is now imposed that the SU(10) ten-dimensional irrep must reduce into the O(3) irreps (0) and (4) and the SU(5) five-dimensional irrep into the O(3) irrep (2). Notice that these two O(3) algebras do not directly coincide with the physical O(3), which nevertheless is contained in their direct product. In figure 1 the structure of the physical chains starting at SU(10) and SU(5) is made explicit. In particular, there are two chains emerging from SU(10).

In an analogous way all remaining cases can be analysed. The complete results are classified in figure 1. From the scheme we see that one can distinguish between nine different dynamical symmetries, i.e. the limits SU(14), O(15), SU(6), SU(5), SU(3), $O(10) \times SU(5)$, $SU(9) \times SU(5)$, $SU(9) \times O(6)$ and $SU(9) \times SU(3)$. The position of the standard IBM scheme inside the extended scheme is shown in the small box in figure 1. As a complement we list in table 1, for each of the occurring algebras, the kind of bosons in which their generators must be realised. It follows that we need to introduce some superscripts to distinguish between similar but unequal algebras.

Table 1. Boson structure of the dynamical symmetry groups and their subgroups.

sdg	dg	sd	sg	d	g
SU(15)	SU(14)	O(6)	SU(10)	$SU(5)^{(b)}$	SU(9)
O(15)	O(14)	$SU(6)^{(b)}$	O(10)	$O(5)^{(b)}$	O(9)
$SU(6)^{(a)}$	$O(5)^{(a)}$	$SU(3)^{(b)}$		$O(3)^{(c)}$	$O(3)^{(b)}$
$SU(5)^{(a)}$	Sp(6)				
$SU(3)^{(a)}$	$O(3)^{(a)}$				

It has been pointed out by Kota (1984) that not all of the dynamical symmetries bear physical relevance. On the other hand, it has to be mentioned that some of the currently proposed (sd)-(g) interaction terms (Van Isacker *et al* 1982), such as the quadrupole-quadrupole interaction, do fit into one of the weak coupling dynamical symmetry limits of the GIBM. From this point of view the type of interaction terms which one adds to a Hamiltonian can be forced to preserve the initial dynamical symmetry of that Hamiltonian.

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